tigated for various angles of attack. The potential flow program required only the shape of the NACA 0012 section and the angle of attack to carry out the calculation. On the average a typical calculation was carried out for 80 points around the airfoil section surface, and required only two seconds of computer time on an IBM 7044 digital computer. All of the velocity and pressure calculations agreed with 2% or better of those presented in Ref. 8. The small discrepancy could be due to the fact that the present method probably used more points on the airfoil surface in the calculations. With these results it is seen that a digital computer can truly make Theodorsen's technique a useful and practical everyday design technique.

With the inviscid velocity and pressure calculated from the potential flow, the boundary-layer flow can be computed. The boundary-layer calculation starts at the stagnation point and proceeds step by step in the x direction until the point of separation (separation is defined as the point where u first becomes negative) is reached, after which it is impossible to proceed further with Eqs. (3) and (4). The results of these calculations are shown in Fig. 2 where the laminar separation point on the upper surface of the NACA 0012 airfoil section has been computed as a function of the angle of attack. The ordinate of the graph is the cordwise distance from the zero angle-of-attack stagnation point, whereas the abscissa is the local angle of attack.

For angles of attack less than 4° or 5° it can be seen from Fig. 2 that the laminar flow calculations are academic, since transition occurs on the airfoil section before laminar separation. However, for angles of attack equal to 50 or greater, laminar separation occurs before transition. Actually, the laminar separation causes the formation of a separation bubble, which plays a significant role in transition and turbulent boundary-layer reattachment. In Ref. 9 measurements on the location of the laminar separation bubble were presented, and it is seen from Fig. 2 that agreement between the theoretical calculations and experiments is excellent. The average boundary-layer calculation took four seconds of computer time on an IBM 7044 computer, and the step size in the η direction was $\Delta\eta = 0.1$. (This corresponds to sixty points across the boundary layer for $\eta \to \infty \simeq 6.0$.) The step size in the x direction was made variable, so that small steps were taken in regions of large pressure variations and large steps in regions of small pressure variation. Also, the experimental location of the laminar separation point should not be sensitive to Reynold's number since the mechanisms for the causation of both the separation bubble and turbulent reattachment are basically inviscid.

Summary and Conclusions

From the previous calculations it was shown that the inviscid flow, pressure distribution and boundary layer flow can be calculated exactly and rapidly over typical airfoil sections. The calculations involve only seconds of computer time on an IBM 7044 computer (not large by current standards), and are capable of giving as much detail as desired about the flowfield. Also, the calculation procedures presented are applicable to arbitrary shaped bodies, and their validity is limited only by the validity of the potential and boundary-layer approximations themselves. Because of the large scale availability of digital computer facilities throughout the world, techniques of calculation like those presented in this paper should have a great impact on the teaching of aerodynamics and on the use of theory in design procedures.

References

[1] Theodorsen, T. and Garrick, I. E., "General Potential Theory of Arbitrary Wing Sections," TR 452, 1933, NACA.

[2] Blottner, F., "Finite Difference Methods of Solution of the Boundary-Layer Equations," *AIAA Journal*, Vol. 8, No. 2, Feb. 1970, pp. 193–205.

[3] Dwyer, H. A., "Solution of a Three-Dimensional Boundary-Layer Flow with Separation," *AIAA Journal*, Vol. 6, No. 7, July 1968, pp. 1336–1342.

[4] Thwaites, B., *Incompressible Aerodynamics*, Oxford University Press, New York, 1960.

[5] Schlichting, H., *Boundary Layer Theory*, 6th ed., McGraw-Hill, New York, 1968.

[6] Richtmyer, R. D. and Morton, K. W., *Difference Methods for Initial Value Problems*, Interscience Publishers, New York, 1st ed., 1957, 2nd ed., 1967.

[7] Von Mises, R., *Theory of Flight*, Dover, New York, 1959.

[8] Abbott, I. H. and von Doenhoff, A. E., *Theory of Wing Sections*, McGraw-Hill, New York, 1949.

[9] McCroskey, W. J. and Dwyer, H. A., "Methods of Analyzing Propeller and Rotor Boundary Layers with Crossflow," *Symposium on Analytical Methods in Aircraft Aerodynamics*, NASA, Oct. 1969.

Ratio of Turbulent Flight Miles to Total Flight Miles in the Altitude Range 45,000–65,000 ft

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Introduction

WHEN an aircraft that is flying through a portion of the atmosphere in which there are no clouds undergoes accelerations that cannot be directly attributable to the movement or setting of the control surfaces or to the flight characteristics of the aircraft, the aircraft is said to be in clear air turbulence. The accelerations of the aircraft are functions of the weight and characteristics of the aircraft in addition to the atmospheric gusts. If the ratio of the turbulent flight miles to total flight miles is to be computed, definitions of turbulent and total flight miles are required. The definition of the turbulent flight miles must include statements that give 1) the lower limit of accelerations that are considered to be turbulence, 2) the frequency interval of interest, and 3) a quantitative evaluation of the duration of turbulence. The ratio may also be a function of 1) the distribution of the total flight miles by season, altitude and underlying terrain, 2) the use and relative success of turbulence search or avoidance procedures, and 3) pattern flying through a known turbulent region.

Discussion and Results

In the High Altitude Clear Air Turbulence (HICAT) program, Crooks, Hoblit, and Prophet[1] defined turbulence as existing if rapid c.g. accelerations in excess of $\pm 0.10\ g$ were observed for a duration of at least 10 sec. In his review of the VGH data from 768,000 miles in the altitude range 40,000–70,000 ft, Steiner[2] defined turbulence to exist "whenever the accelerometer trace was disturbed and contained gust velocities (presumably 'derived' gust velocities) greater than 2 fps." A comparison of the HICAT deived gust velocities with those given by Steiner indicates that the two definitions are roughly equivalent. Crooks et al.,[1] and Ashburn, Waco, and Melvin[3] also defined turbulence in terms of the rms gust velocity. This definition is not useful for determining the ratio of turbulent flight miles to total flight miles because the rms gust velocity data are not available for all the turbulent regions observed.

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Table 1 Ratio of turbulent flight miles to total flight miles

Category, ft	Total flight miles	Ratio turbulent
65,000–69,900	16,000	0.019
60,000–64,900	87,000	0.014
55,000–59,900	107,000	0.028
50,000–54,900	105,000	0.047
45,000–49,900	30,000	0.042
45,000–69,900	345,000	0.031
Water	124,000	0.027
Flatland	132,000	0.027
Low mountains	49,000	0.036
High mountains	40,000	0.049
Winter (Dec.–Feb.)	108,000	0.038
Spring (March–May)	81,000	0.039
Summer (June–Aug.)	71,000	0.019
Autumn (Sept.–Nov.)	85,000	0.024

Efforts were made during the HICAT flight program to seek turbulent regions and to fly repeatedly through these regions once they were located. Thus, the HICAT records presumably indicate a higher proportion of turbulence than would be found on random flights. In the results presented below, adjustments were made to correct for the effects of repeated flights through the turbulent regions. Table 1 lists the ratios of turbulent flight miles to total flight miles for the HICAT sample. Flatland was defined as land area with relief differences <3000 ft. The analogous relief differences for low mountains and high mountains were 3000–7000 ft and >7000 ft, respectively. The percentage of HICAT flight miles that were turbulent (Table 1) were judged to be high, even with the effect of pattern flights within turbulent regions removed, because of the apparent skill in locating turbulent areas. Table 3 presents a subjective estimate of representative values of percentage of flight miles expected to be turbulent. This estimate was based on the results given in Tables 1 and 2.

Table 2 Comparison of Steiner (NASA U-2), and HICAT data

Altitude interval, ft	Steiner (1966) Flight miles	Steiner (1966) Ratio turbulent	HICAT Flight miles	HICAT Ratio turbulent
60,000–70,000	576,000	0.006	103,000	0.015
50,000–60,000	141,700	0.020	212,000	0.035
40,000–50,000	49,500	0.025	30,000	0.042
40,000–70,000	768,100	0.009	345,000	0.031

Table 3 Recommended ratio of turbulent flight miles to total flight miles

Category, ft	Ratio turbulent
65,000–70,000	0.010
60,000–65,000	0.010
55,000–60,000	0.020
50,000–55,000	0.030
45,000–50,000	0.030
45,000–70,000	0.020
Water	0.018
Flatland	0.018
Low mountains	0.022
High mountains	0.030
Winter	0.024
Spring	0.024
Summer	0.014
Autumn	0.018

Concluding Remarks

Based on data from more than 200 flights the percentage of flight miles that were turbulent has been shown to increase with underlying terrain roughness, decrease with altitude, and display a maximum in the winter and spring. The sample size in several of the underlying terrain, altitude, and seasonal categories limited definite conclusions as to which of these factors most strongly influenced the distribution of turbulence. However, a more detailed investigation which restricted comparisons to categories with a large number of flight miles has shown the following: 1) Turbulence over mountains was definitely more pronounced in magnitude and extent during the winter season than over flat terrain. 2) The amount and intensity of turbulence decreased with altitude only slightly between 45,000 and 65,0000 ft over mountains but quite rapidly over flat terrain.

References

[1] Crooks, W. M., Hoblit, F. M., and Prophet, D. T., "Project HICAT. An Investigation of High Altitude Clear Air Turbulence," AFFDL-TR-67-123, Nov. 1967, Air Force Flight Dynamics Lab., Wright-Patterson Air Force Base, Ohio.

[2] Steiner, R., "A Review of NASA High Altitude Clear Air Turbulence Sampling Programs," *Journal of Aircraft*, Vol. 3, No. 1, Jan.-Feb. 1966, pp. 48-52.

[3] Ashburn, E. V., Waco, D. E., and Melvin, C. A., "High Altitude Gust Criteria for Aircraft Design," Sept. 1970, Air Force Flight Dynamics Lab., Wright-Patterson Air Force Base, Ohio, (in press).